Summary of Chapter 2

The von Mangoldt function, Λ, is introduced as the coefficients in the Dirichlet series for $-\zeta'(s)/\zeta(s)$. Its fundamental property is that

$$\sum_{d|n} \Lambda(d) = \log n,$$

for all $n \geq 1$.

By looking at Euler products it is shown that Λ is non-zero only on prime powers so we define $\psi(x) = \sum_{n \leq x} \Lambda(n)$ as well as $\pi(x) = \sum_{p \leq x} 1$.

Importantly we prove Chebyshev's results that

$$ax < \psi(x) < bx \quad \text{and} \quad \frac{ax}{\log x} < \pi(x) < \frac{bx}{\log x}$$

for any $a < \log 2$, $b > 2\log 2$, with x sufficiently large.

The fundamental technique of Partial Summation is introduced. This allows the removal or introduction of weights into sums. As an example it is used to prove Merten's result

$$\sum_{p \leq x} \frac{1}{p} = \log\log x + O(1),$$

which improves a lower bound result of the same form given in Chapter 1.

Finally the Prime Number Theorem in the form

$$\pi(x) \sim \frac{x}{\log x}$$

is discussed. It is shown that it is equivalent to $\psi(x) \sim x$.